DOCUMENT RESUME

ED 410 277 TM 027 087

AUTHOR Wang, Jianjun
TITLE Demystification of Hierarchical Linear Model Using SAS PROC MIXED.
SPONS AGENCY California State Univ., Bakersfield.
PUB DATE Mar 97
NOTE 21p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, March 24-28, 1997).
PUB TYPE Numerical/Quantitative Data (110) -- Reports - Evaluative (142) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Computer Software; *Data Analysis; *Educational Research; *Research Methodology; Tables (Data)
IDENTIFIERS *Hierarchical Linear Modeling; *Statistical Analysis System

ABSTRACT

Hierarchical data analyses in different discipline areas are reviewed in this article to compare statistical applications between the Hierarchical Linear Model (HLM) software and the Statistical Analysis System (SAS) MIXED procedure. Similar features of the two software programs are illustrated through reconfirmation of an HLM example using the MIXED procedure. Because SAS is a standard statistical package with a much larger group of users, discussions of the shared features in statistical computing may present additional options to demystify the existing methods for hierarchical data analyses. It is concluded that the SAS program can reproduce most results of HLM. Comparison also uncovers some potential "p" value computation problems with HLM. An appendix presents an SAS program to confirm an HLM example. (Contains 5 tables and 21 references.) (Author/SLD)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

Running head: HLM & PROC MIXED

ED 410 277

Demystification of Hierarchical Linear Model Using SAS PROC MIXED*

Jianjun Wang
Department of Teacher Education
California State University, Bakersfield
9001 Stockdale Highway
Bakersfield, CA 93311

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

Jianjun Wang

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

TM027087

* This research was supported by a research grant from the Faculty Diversity Grant Program of California State University, Bakersfield during the 1996-97 academic year.

Paper presented at the 1997 American Educational Research Association (AERA) annual meeting, March 24-28, Chicago, IL.

BEST COPY AVAILABLE

Abstract

Hierarchical data analyses in different discipline areas are reviewed in this article to compare statistical applications between the Hierarchical Linear Model (HLM) software and the SAS MIXED procedure. Similar features of the two software are illustrated through reconfirmation of an HLM example using the MIXED procedure. Because SAS is a standard statistical package with a much larger group of users, discussions of the shared features in statistical computing may present additional options to demystify the existing methods for hierarchical data analyses.

Demystification of Hierarchical Linear Model Using SAS PROC MIXED

Traditional statistical methods, such as multiple regressions and analyses of variance, are based on general linear models (Graybill, 1976; Milliken & Johnson, 1984). Assumptions associated with these methods are rarely met in many educational studies. Raudenbush (1988), for instance, observed:

> The traditional linear models on which most researchers rely require the assumption that subjects respond independently to educational programs. In fact, subjects are commonly "nested" within classrooms, schools, districts, or program sites so that responses within groups are dependent. (p. 85)

In the cases that the traditional methods no longer fit, the research community is left with two options, developing a new method or introducing appropriate methods from other disciplines.

Among several software packages recently developed for hierarchical data analyses, de Leeuw (1992) noted that "The program HLM [Hierarchical Linear Model], by Bryk and Raudenbush, was the friendliest and most polished of these products, and in rapid succession a number of convincing and interesting examples were published" (p. xv). Meanwhile, two authors of an HLM book acknowledged:

> The models discussed in this book appear in diverse literatures under a variety of titles. In sociological research, they are often referred to as multilevel linear models (cf. Goldstein, 1987; Mason et al., 1983). In biometric applications, the terms mixed-effects models and random-effects models are common (cf. Elston & Grizzle, 1962; Laird & Ware, 1982). (Bryk & Raudenbush, 1992, p. 3)

Nonetheless, not all researchers in social sciences are well informed about the recent development of research methodology, such as the mixed-effects modeling, in other disciplines. James (1995), for instance, was unaware of the MIXED procedure in SAS, and

claimed that "The estimation procedures for Hierarchical Linear Models is not available in any standard statistical packages such as SPSS or SAS" (p. 1). Perhaps this was in part because "mixed model methodology has advanced rapidly in recent years, and even statisticians who studied the topic five years ago may not be aware of the tremendous new capabilities available for applications of mixed models" (Littell, Milliken, Stroup, & Wolfinger, 1996, p. vii). The purpose of this study is to review and illustrate the area of parallel applications between the hierarchical linear model (HLM) and the mixed model methodology in SAS. Because the MIXED procedure in SAS supports a syntax similar to the one used in the General Linear Model (GLM) procedure, researchers who had some experience in GLM are unlikely having a sense of mystification with the MIXED procedure. In addition, because standard statistical packages like SAS are available to most data analysts, the demystification may benefit other HLM users who need additional options in hierarchical data analyses.

Background

Both HLM and the MIXED procedure share similar areas of application. In the area of hierarchical linear modeling, de Leeuw and Kreft (1995) pointed out:

> Hierarchical data occur if the objects we study are classified into groups. Students within classes is one classical example; individual in census tracts or political districts in another one; and time points within individual is a third one. (p. 172)

These applications also fit well in the two most typical scenarios of using the MIXED procedure. According to SAS (1996),

> The first scenario can be generalized to include one set of clusters nested within another. For example, if students are the experimental unit, they can be clustered into

> classes, which in turn can be clustered into schools. Each level of this hierarchy can introduce an additional source of variability and correlation. The second scenario occurs in longitudinal studies, where repeated measurements are taken over time. (p. 534)

In a hierarchical data structure, factors at each level could be random or fixed (Wolfinger, 1992), depending on the research design. Definitions of random and fixed factors are documented in many statistics textbooks (e.g., Casella & Berger, 1990; Littell, Milliken, Stroup, & Wolfinger, 1996; Littell, Freund, & Spector, 1991; Milliken & Johnson, 1984; Ott, 1993; Stevens, 1990). Casella and Berger (1990) wrote:

> A factor is a *fixed factor* if all the values of interest are included in the experiment. A factor is a *random factor* if all the values of interest are *not* included in the experiment and those that are can be considered to be randomly chosen from all the values of interest. (p. 529)

Milliken and Johnson (1984) noted that "A model is called a *mixed* or *mixed effects model* if some of the factors in the treatment structure are fixed effects and some are random effects" (p. 213).

In general, mixed models include both fixed and random models as special cases (Ott, 1993). Bryk, Raudenbush, Seltzer, and Congdon (1989) further clarified:

> In the language of variance, the hierarchical linear model can be viewed as a mixed model: the within-unit parameters, β_j, are random (as is R_j), and the between-unit parameters, γ, are fixed. In fact, any of the with-in parameters may also be treated as fixed. That is, a common β coefficient can be estimated for all groups rather than assuming a different value for each group. (p. 10)

Applications of the mixed model methodology can be dated back to at least the Yates' (1937) split-plot designs. Cochran and Cox (1957) elaborated this type of design in agronomy:

> In field experiments an extra factor is sometimes introduced into an experiment by dividing each plot into a number of parts. (p. 293)

> In the statistical analysis, account must be taken of the fact that the observations from different sub-units in the same unit may be correlated. In field experiments this correlation is just a reflection of the fact that neighboring pieces of land tend to be similar in fertility and in other agronomic properties. (p. 294)

The hierarchical structure of split-plot design is parallel to the educational circumstance with students nested in classes sharing similar characteristics of instruction and academic performance.

In contrast, some substantial differences also exist between the split-plot experiment and the sampling or observational studies in education. Data in a split-plot design can be collected from different levels of unit according to a pre-designed research plan. Given the choice, Kirk (1968) compared advantages between the split-plot design and a factorial design, and concluded that the split-plot design was more powerful for detecting sub-plot effects. "On the other hand, if equal precision for all treatment effects is desired, the average power of a type RBF-pq [pxq randomized block factorial design] design is greater" (Kirk, 1968, p. 317). Littell, Freund, and Spector (1991) concurred:

> A split-plot design is also useful when more information is needed for comparing the levels of one factor than for comparing the levels of the other factor. In this case, the factor for which more information is needed should be the subplot factor. (p. 130)

In the education community, few researchers addressed the unequal powers of detecting statistical effects at different levels of measurement (e.g., Bryk & Raudenbush, 1992). This observation could be resulted from a special consideration over the nested feature of school system which sets the hierarchical measurement beyond the researchers' control. Consequently, the keywords in agricultural statistics are design, ANOVA, and variance components, while in educational statistics the keywords are observational study, regression,

and random coefficients (de Leeuw, personal communications, July 25, 1996). Despite the fact that the hierarchy was not an option of choice in most educational studies, the research community ought to be alerted that the statistical analysis at lower levels is more informative than that of higher levels.

In short, the mixed model methodology is needed in educational research, as well as in other subject areas. Although different names were given to such methods, similarities among the multilevel designs present a strong rationale for researchers of different subjects to share information in empirical applications. The subsequent discussion of a specific example illustrates similar results between the HLM software and the SAS MIXED procedure. The result comparison was not intended to convert researchers from their existing HLM or SAS user tracks, but to inform the research community that features of either software can be selectively chosen to pursue satisfactory findings.

Reconfirmation of an HLM Example Using the SAS MIXED Procedure

A wide variety of applications in the area of hierarchical linear modeling have resulted in a further development of the mixed model methodology in the recent years (Wolfinger, 1992). Littell, Milliken, Stroup, and Wolfinger (1996) pointed out that "Most of the mixed model advancements in the SAS System are in the MIXED procedure" (p. vii). In Appendix 1, the MIXED procedure was employed to reconfirm an example of hierarchical data analysis in an HLM user's guide (Bryk, Raudenbush, Seltzer, & Congdon, 1989). While both HLM and SAS are fairly easy to use, the confirmation of an HLM example using the SAS MIXED procedure might meet more researchers' interest because SAS is a standard statistical software

with a much larger group of users.

The simplest hierarchical linear model involves two levels of data structure. In the second version of the HLM user's guide (Bryk, Raudenbush, Seltzer, & Congdon, 1989), a "rat data" was analyzed as the first example to illustrate the two level modeling. The user's guide contains the following example introduction:

> The first is the "rat data" which has been analyzed by a number of investigators. The first use of these data with HLM appeared in Strnio, Weisberg, and Bryk, 1983. The data consist of the weights of ten rats at birth and after each of four subsequent weeks. Although the data were collected as part of an experiment, only the control group information is used here. In addition, we have one between-unit variable - the weight of each rat's mother - that can be used in the between-unit model. This simple data set provides a useful illustration of an HLM application in assessing change. (Bryk, Raudenbush, Seltzer, and Congdon, 1989, p. 12)

In version 4 of the HLM user's guide (Bryk, Raudenbush, & Congdon, 1996), the rat data were replaced by the High School and Beyond (HS&B) data from the National Center for Education Statistics (NCES). Although using a national data set, such as the HS&B, can demonstrate the capability of HLM in handling large scale data bases, a recent policy of the Department of Education requires a restricted data license to access most NCES data bases, including HS&B. Because the requirement includes the General Attorney's endorsement from each state (NCES, 1996), not all researchers have the opportunity to access the HS&B data. In this article, the purpose of example reconfirmation is to illustrate that additional tools, such as SAS, can be used for the hierarchical data analysis, rather than demonstrating the HLM capability for large data analyses. Accordingly, the well disseminated rat data base, which is accessible to all researchers (Strnio, Weisberg, & Bryk, 1983), is a better example choice for the result reconfirmation.

The rat data contain repeated weight measurement of 10 rats over a four week period (weeks 0, 1, 2, 3, and 4), as well as the weight of the rat's mother. Let Y_{it} denote the weight of the ith rat at the tth week. The model employed by Strenio, Weisberg and Bryk (1983) are:

$$E(Y_{it} \mid t) = \pi_{1i} + \pi_{2i} t \quad (1)$$

$$E(\pi_{1i} \mid x_i) = \gamma_{11} + \gamma_{12} x_i \quad (2)$$

and

$$E(\pi_{2i} \mid x_i) = \gamma_{21} + \gamma_{22} x_i \quad (3)$$

where x_i represents mother's weight for rat i, $i = 1, \ldots, 10$, and $t = 0, \ldots, 4$.

In the HLM terminology, the γ coefficients specify the effects of between-unit factors, such as the mother's weight, on the rat's growth trajectories. The HLM software estimates the growth trajectories for each rat, the γ coefficients, as well as the estimated covariance parameters for the random effects (Bryk, Raudenbush, Seltzer, & Congdon, 1989). This individual growth model also fits in one of the application scenarios with the MIXED procedure (SAS, 1996).

The estimates of the rats' growth trajectories were copied from the HLM user's guide (Bryk, Raudenbush, Seltzer, & Congdon, 1989, p. 23) to Table 1. Table 2 contains similar estimates produced by the SAS MIXED procedure. It is clear that SAS not only reproduced the HLM results, but provided standard errors and t statistics for the estimates related to the rats' growth trajectories.

Tables 1 & 2 inserted around here

The SAS program also produced the estimates of γ, standard errors, and t statistics (Table 3). These results generally match the findings on page 28 of the HLM user's guide (Bryk, Raudenbush, Seltzer, & Congdon, 1989).

Table 3 inserted around here

For a correlated random coefficient model in equations (1), (2) and (3), an unstructured covariance structure is recommended by SAS (1996) to accomondate different variances of the intercept and slope, as well as the covariance between them. The covariance parameters estimated by the SAS program (Appendix 1) reconfirmed the corresponding result of HLM (Table 4). Inspection of Table 4 further revealed that different statistical tests were employed in HLM and SAS. The Z test results in the SAS printout are based on an asymptotic feature which is more applicable to large data analyses (Littell, Milliken, Stroup, & Wolfinger, 1996). In this respect, HLM could be a better choice when the sample size is small.

Table 4 inserted around here

On the other hand, two p values reported in the t tests of γ estimates are copied from page 28 of Bryk, Raudenbush, Seltzer, and Congdon (1989) to Table 5. Because the rat data contain a total of 60 observations, the degree of freedom for any t tests should be no larger than 60. Under the condition of df $\leq$60 and t = .899, the p value should be between .372 and .534. Similarly, for df $\leq$60 and t = .250, the p value is between .803 and .844. Accordingly,

the p values in Table 5 are beyond the corresponding ranges, and thus, cannot be reconfirmed through SAS.

Table 5 inserted around here

In summary, the SAS program in Appendix 1 can be employed to reproduce most results of HLM. This comparison also uncovers some potential p value computation problems with HLM.

The HLM software was first released in the late 1980s while the MIXED procedure was not disseminated until 1992. Perhaps due to the time difference, considerations were taken by the mixed model researchers to include the HLM applications as special cases. One of the most useful mixed model books co-authored by Littell, Milliken, Stroup, and Wolfinger (1996) devoted a whole chapter on the HLM cases. The authors noted:

> Data that have a nested or hierarchical structure are common in a wide variety of disciplines, and similar methods for analyzing such data are found in these disciplines under different guises. The analyses considered here fall under the headings of **random coefficient models** and **empirical Bayes models** in the statistics literature (Laired and Ware, 1982; Strenio, Weisberg, and Bryk, 1983; Rutter and Elashoff, 1994; Wolfinger, 1996). Analogous terms in the educational and social science literature are **hierarchical linear models** and **multilevel linear models** (Goldstein, 1987; Bryk and Raudenbush, 1992; see also *Journal of Educational and Behavioral Statistics* (1995), **20**(2)). A primary objective of this chapter is to describe these models and illustrate how to fit them using PROC MIXED. (Littell, Milliken, Stroup, & Wolfinger, 1996, p. 253)

Although Strenio, Weisberg, and Bryk's (1983) article which contains the rat data was cited in the above paragraph, no examples of HLM (Bryk, Raudenbush, & Congdon, 1996;

Bryk, Raudenbush, Seltzer, & Congdon, 1989) were thoroughly discussed in the mixed model documentation. By reviewing and demonstrating the areas of overlap between HLM and the SAS MIXED procedure, this article may serve the purpose of demystifying the application of SAS in hierarchical data analyses.

References

Bryk, A. S. & Raudenbush, S. W. (1992). Hierarchical linear models: Applications and data analysis methods. London, UK: Sage Publications.

Bryk, A., Raudenbush, S., & Congdon, R. (1996). Hierarchical linear and nonlinear modeling with the HLM/2L and HLM/3L programs. Chicago, IL: Scientific Software International.

Bryk, A., Raudenbush, S., Seltzer, M., & Congdon, R. (1989). An introduction to HLM: Computer program and user's guide. Chicago, IL: Scientific Software International.

Casella, G and Berger, R. L. (1990). Statistical inference. Pacific Grove, CA: Brooks/Cole Publishing.

Cochran, W. G., & Cox, G. M. (1957). Experimental designs (2nd ed). New York, NY: John Wiley & Sons.

de Leeuw, J. (1992). Series editor's introduction to hierarchical linear models. In A. Bryk, & S. Raudenbush, Hierarchical linear models: Applications and data analysis methods. London, UK: Sage Publications.

de Leeuw, J. & Kreft, I. G. G. (1995). Questioning multilevel models. Journal of Educational and Behavioral Statistics, 20 (2), 171-189.

Graybill, F. A. (1976). Theory and application of the linear model. Pacific Grove, CA: Wadsworth & Brooks/Cole Advanced Books & Software.

James, D. R. (1995). Hierarchical linear models: Statistical estimation and interpretation. (World Wide Web address: http://infotech.indiana.edu/stf/sociology.html)

Kirk, R. E. (1968). Experimental design: procedures for the behavioral sciences.

ST COPY AVAILABLE

Belmont, CA: Brooks/Cole.

Littell, R., Freund, R., & Spector, P. (1991). SAS system for linear models, (3rd ed.). Cary, NC: SAS Institute.

Littell, R. C., Milliken, G. A., Stroup, W. W., & Wolfinger, R. D. (1996). SAS system for mixed models. Cary, NC: SAS Institute.

Milliken, G. A. & Johnson, D. E. (1984). Analysis of messy data, Volume 1: Designed Experiments. New York, NY: Chapman and Hall.

NCES (1996). Restricted-use data procedures manual (NCES 96-860). Washington, DC: U.S. Department of Education.

Ott, R. (1993). An introduction to statistical methods and data analysis (4th ed.). Belmont, CA: Duxbury.

Raudenbush, S. W. (1988). Educational applications of hierarchical linear models: A review. Journal of Educational Statistics, 13 (2), 85-116.

SAS (1996). SAS/STAT software changes and enhancements through release 6.11. Cary, NC: SAS Institute.

Stevens, J. (1990). Intermediate statistics -- A modern approach. Hillsdale, NJ: Lawrence Erlbaum.

Strnio, J., Weisberg, H., & Bryk, A. (1983). Empirical Bayes estimation of individual growth-curve parameters and their relationship to covariates. Biometrics, 39, 71-86.

Wolfinger, R. (1992). A tutorial on mixed models. Cary, NC: SAS institute.

Yates, F. (1937). The design and analysis of factorial experiments. Harpenden, England: Imperial Bureau of Soil Science.

Table 1

The HLM estimates of parameters for the rats' growth trajectories

HLM Printout

[p. 23 of Bryk, Raudenbush, Seltzer, and Congdon (1989)]

WITHIN-UNIT REGRESSIONS

UNIT		INTERCEPT	WEEK
ID = 1	, THE BETA_HAT =	111.40000	28.80000
ID = 2	, THE BETA_HAT =	120.20000	28.10000
ID = 3	, THE BETA_HAT =	119.80000	36.30000
ID = 4	, THE BETA_HAT =	103.40000	27.20000
ID = 5	, THE BETA_HAT =	100.00000	23.40000
ID = 6	, THE BETA_HAT =	99.00000	29.30000
ID = 7	, THE BETA_HAT =	93.00000	25.60000
ID = 8	, THE BETA_HAT =	113.60000	19.70000
ID = 9	, THE BETA_HAT =	90.40000	23.60000
ID = 10	, THE BETA_HAT =	121.00000	25.60000

Table 2

The SAS estimates of parameters for the rats' growth trajectories

SAS Printout

[p. 1 of the listing file created by the SAS program in Appendix 1]

Solution for Fixed Effects

Parameter	Estimate	Std Error	DDF	T	Pr > |T|
RAT 1	111.40000000	4.28361218	30	26.01	0.0001
RAT 2	120.20000000	4.28361218	30	28.06	0.0001
RAT 3	119.80000000	4.28361218	30	27.97	0.0001
RAT 4	103.40000000	4.28361218	30	24.14	0.0001
RAT 5	100.00000000	4.28361218	30	23.34	0.0001
RAT 6	99.00000000	4.28361218	30	23.11	0.0001
RAT 7	93.00000000	4.28361218	30	21.71	0.0001
RAT 8	113.60000000	4.28361218	30	26.52	0.0001
RAT 9	90.40000000	4.28361218	30	21.10	0.0001
RAT 10	121.00000000	4.28361218	30	28.25	0.0001
T*RAT 1	28.80000000	3.02897122	30	9.51	0.0001
T*RAT 2	28.10000000	3.02897122	30	9.28	0.0001
T*RAT 3	36.30000000	3.02897122	30	11.98	0.0001
T*RAT 4	27.20000000	3.02897122	30	8.98	0.0001
T*RAT 5	23.40000000	3.02897122	30	7.73	0.0001
T*RAT 6	29.30000000	3.02897122	30	9.67	0.0001
T*RAT 7	25.60000000	3.02897122	30	8.45	0.0001
T*RAT 8	19.70000000	3.02897122	30	6.50	0.0001
T*RAT 9	23.60000000	3.02897122	30	7.79	0.0001
T*RAT 10	25.60000000	3.02897122	30	8.45	0.0001

Table 3

The gamma (γ) statistics, standard errors, and t statistics produced by HLM and SAS

HLM Results

[p. 28 of Bryk, Raudenbush, Seltzer, and Congdon (1989)]

THE GAMMA(*)-STANDARD ERROR-T STATISTIC TABLE:

		GAMMA(*)	STANDARD ERROR	T STATISTIC
FOR	BASE	*<Results for the between-unit equation in which the outcome variable is the within-unit intercept>*		
	BASE	18.873660	21.002153	.899
	MOMWGHT	.545101	.128963	4.227
FOR	WEEK	*<Results for the between-unit equation in which the outcome variable is the within-unit slope for the WEEK variable>*		
	BASE	2.967709	11.848147	.250
	MOMWGHT	.146866	.072753	2.019

SAS Results

[p. 4 of the listing file created by the SAS program in Appendix 1]

Solution for Fixed Effects

Parameter	Estimate	Std Error	DDF	T	Pr > |T|
INTERCEPT	18.87365994	21.00215302	8	0.90	0.3951
T	2.96770893	11.84814661	8	0.25	0.8085
MOM	0.54510086	0.12896265	30	4.23	0.0002
T*MOM	0.14686599	0.07275294	30	2.02	0.0525

Table 4

The estimated covariance parameters using HLM and SAS

HLM Results

[p. 28 of Bryk, Raudenbush, Seltzer, and Congdon (1989)]

THE CHI SQUARE TABLE:

PARAMETER	ESTIMATED PARAMETER VARIANCE	DEGREES OF FREEDOM	CHI SQUARE	P-VALUE
BASE COEF.	27.81800	8	20.127	.010
WEEK SLOPE	5.51801	8	12.811	.118

SAS Results

[p. 3-4 of the listing file created by the SAS program in Appendix 1]

Covariance Parameter Estimates (REML)

Cov Parm		Ratio	Estimate	Std Error	Z	Pr > |Z|
INTERCEPT	UN(1,1)	0.30321906	27.81933814	23.56550640	1.18	0.2378
	UN(2,1)	-0.08990193	-8.24820281	9.65922434	-0.85	0.3931
	UN(2,2)	0.06015123	5.51867510	7.71914548	0.71	0.4747
Residual		1.00000000	91.74666667	23.68888747	3.87	0.0001

Table 5

Qusetionable p values produced by HLM in t tests over the estimated γ parameters*

Examples	Gamma	t	p
<Results for the between-unit equation in which the outcome variable is the within-unit intercept>			
Base	18.873660	.899	.245
<Results for the between-unit equation in which the outcome variable is the within-unit slope for the WEEK variable>			
Base	2.967709	.250	.369

* This table was constructed according to the information on page 28 of Bryk, Raudenbush, Seltzer, and Congdon (1989).

Appendix 1: A SAS Program to Confirm A HLM Rat Example

```
/* The example is taken from Bryk, A., Raudenbush, S., Seltzer, M., & Congdon, R. (1989).
"An introduction to HLM: Computer program and user's guide". The full data set was
presented in an article of Strnio, J., Weisberg, H., & Bryk, A. (1983) published in
"Biometrics", vol. 39, pages 71-86. The data contain repeated weight measurements for 10
rats over a 4 week period (weeks 0, 1, 2, 3, and 4) and weight of the rat's mother.*/

/* Data explanation: The first 5 variables are the rats' weights at the five time points and the
last variable is the moms' weight. */

options ls=64 ps=53;
DATA A (KEEP=RAT T W MOM);
 INPUT Y1-Y5 MOM;
 ARRAY Y{5} Y1-Y5;
 RAT = _N_;                    /* rat id variable */
 DO TIME=1 TO 5;
   T=TIME-3;                   /* center the times at 0 */
   W=Y{TIME};
   OUTPUT;
 END;
CARDS;
61 72 118 130 176 170
65 85 129 148 174 194
57 68 130 143 201 187
46 74 116 124 157 156
47 85 103 117 148 155
43 58 109 133 152 150
53 62  82 112 156 138
72 96 117 129 154 154
53 54  87 120 138 149
72 98 114 144 177 167
;
/* First, regress the rats' weight on time.*/
PROC MIXED;
 CLASS RAT;
 MODEL W=RAT RAT*T / NOINT S;          /* rat's growth trajectories */

/* This program provides all other information */
PROC MIXED;
 CLASS RAT;
 MODEL W=T MOM T*MOM/S;
 RANDOM INT T/TYPE=UN SUB=RAT GCORR G;
run;
```